Emerging auto-oscillations in a non-linear dynamical model of multiple feedbacks in soil

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How macro-patterns of the living systems behavior arise from the dynamics on micro-scale is a crucial phenomena of self-organization in complex systems. In this study we mathematically model a soil ecosystem with multiple feedbacks and strongly non-linear behavior. The model includes microbial growth and biotic/abiotic/autocatalytic organic matter transformations affected by oxygen, water and capillary pore size.

The mathematical model is formulated as a system of parabolic type non-linear partial differential equations with the elements of a discrete particles method. The energy of external impact on a system is defined by a constant oxygen level on the boundary. The system of partial differential equations is solved using second order numerical scheme with iterations of a matrix form of Thomas algorithm, time step auto-selection and accuracy control. To find the auto-oscillating regime of the system the coefficients were analytically pre-evaluated in a simplified system where the Hopf-bifurcation was found. Then the full system was solved numerically tracking the occurrence of the auto-oscillating regime.

Initially homogeneous distributions of the system state variables evolve into spatially heterogeneous stable dynamical structures. Non-linear as opposed to linear oxygen diffusion shows possibility of modeling anaerobic micro-zones formation (organic matter conservation mechanism).

The results visualize the system's flashing evolution patterns of interactions between alive, organic components and physical environmental factors. We show the occurrence of auto-oscillating regime and the existence of irregular attractor (in projections of the statistical phase portrait onto 2D subspaces) in which attraction basin the system structures are dynamically stable/self-organized. The integral of the CO_2 state variable is used as a macroscopic parameter to describe system as a whole and validate on experimental data of soil heterotrophic respiration (temperature and moisture series). The model is used to search for relations between stability of microbial community and the existence of stable attractors in complex dynamical systems.